BETTER SOLUTION TO PAT’S PROBLEM

STEVE

Pat’s two equations:

\[(1) \quad \frac{a(r^6 - 1)}{r - 1} = A = 32327011.49 \cdot 6\]
\[(2) \quad \frac{a(r^{18} - 1)}{r - 1} = B = 32327011.49 \cdot 6 + 51065497.13 \cdot 12\]

Divide (2) by (1):

\[(3) \quad \frac{r^{18} - 1}{r^6 - 1} = \frac{B}{A}\]

Simplify the left-hand side:

\[(4) \quad 1 + r^6 + r^{12} = \frac{B}{A}\]

Let \(x = r^6\):

\[(5) \quad 1 + x + x^2 = \frac{B}{A}\]

Rewrite slightly:

\[(6) \quad x^2 + x + \left(1 - \frac{B}{A}\right) = 0\]

Solve (6) using the quadratic formula:

\[(7) \quad x = \frac{-1 \pm \sqrt{1 - 4 \left(1 - \frac{B}{A}\right)}}{2}\]

Use a calculator to find \(\frac{B}{A} = 4.159308255\). Substitute into (7):

\[(8) \quad x = \frac{-1 \pm \sqrt{1 - 4 \left(1 - 4.159308255\right)}}{2} = \frac{-1 \pm \sqrt{13.637233020}}{2}\]

Use the positive square root so we don’t get a negative number:

\[(9) \quad x = \frac{-1 + \sqrt{13.637233020}}{2} = 1.346431221\]

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Since $r^6 = x$,

\[(10) \quad r = (1.346431221)^{1/6} = 1.050825723\]

Thus the growth rate is 5% as before, and from (1),

\[(11) \quad a = A \frac{r - 1}{r^6 - 1} = 32327011.49 \cdot 6 \cdot \frac{1.050825723 - 1}{1.050825723^{6} - 1} = 28456622.66.\]

Thus first month’s take is $28,456,622.66$.

This method explains why Maple found exactly 12 answers including complex ones. Equation (8) gives two values for $x$. Since $r^6 = x$ and every nonzero number has exactly six sixth roots over the complex numbers, there are twelve possible values for $r$. 